Stopped Muon Decay Lifetime Shifts due to Condensed Matter

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Up to second order in $\alpha = (e^2/\hbar c)$, vacuum electro-magnetic corrections to weak interaction induced charged particle lifetimes have been previously studied. In the laboratory, stopped muon lifetimes are measured in a condensed matter medium whose radiation impedance differs from that of the vacuum. The resulting condensed matter corrections to first order in α dominate those vacuum radiative corrections (two photon loops) which are second order in α.

For unstable charged particles, such as the muon, electro-magnetic corrections to the lifetime are essential for precise determinations of weak interaction coupling strengths. For example, tests of universality for lepton couplings to the weak current are meaningful only after radiative corrections have been applied. The needed calculations have a long history [1–3], and investigations into the limit $(m_e/M_\mu) \to 0^+$ were quite fruitful. These investigations eventually led to wonderful insights into the nature of mass singularities, e.g. the Kinoshita-Lee-Nauenberg theorem [4,5], and their applications to jet definitions [6,7]. More recently, computations of two-loop photon corrections to second order in $\alpha = (e^2/\hbar c)$ have also been made [8].

All of the above calculations are applicable to muons which decay in the vacuum. In *real experiments*, the muons are stopped in condensed matter before they decay. Curiously, and even though condensed matter environment related life time shifts of decaying nuclei have been experimentally confirmed [9], we have been unable to find similar studies for weak charged particle decays. Such an effect must exist since the radiation spectrum (emitted during a charged particle decay) depends on the nature of material environment. In what follows, we shall exhibit the radiative corrections to the weak decay rate for muons both in condensed matter and in the vacuum. We conclude, for *laboratory experiments*, that condensed matter effects to first order in α dominate those vacuum effects which are of second order in α.

To lowest order in electro-weak theory, the Fermi transition rate Γ_F for the muon decay $\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$ yields

$$\Gamma_F = \left(\frac{M_\mu c^2}{192\pi^3 \hbar}\right)\left(\frac{G_F M_\mu^2}{\hbar c}\right)^2 \left\{1 - 8\left(\frac{m_e}{M_\mu}\right)^2\right\}. \quad (1)$$

The first Feynman diagram of FIG.1 corresponds to Eq.(1) The radiative corrections to lowest order in α are described by the five remaining Feynman diagrams.

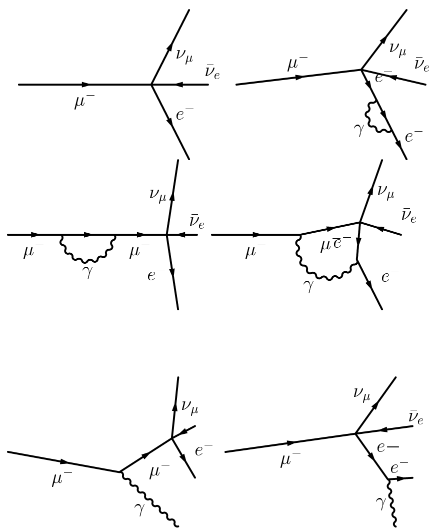

FIG. 1. Shown are (i) the Fermi interaction diagram for muon decay as in Eq.(1), (ii) the electron wave function renormalization to first order in α, (iii) the muon wave function renormalization to first order in α, and (iv) the vertex renormalization to first order in α. In these diagrams, photons are virtual. Shown in (v) and (vi) are the "real" Bremsstrahlung photon emission diagrams, respectively, from the muon and electron.

One computes from the amplitudes in FIG.1, the radiative corrections to Γ_F to lowest order in α. The final answer for the total decay rate is

$$\Gamma_{tot} = \Gamma_F \left\{1 - \left(\frac{\alpha}{2\pi}\right)\left(\pi^2 - \left(\frac{25}{4}\right) + \eta\right) + \ ...\right\}, \quad (2)$$

where the parameter η will be used to describe the condensed matter effects.

For the vacuum case with $\eta = 0$, Eq.(2) is well known. The vacuum radiative corrections slightly suppress the muon decay rate. The physical reason is that the virtual photon diagrams (when interfering with the zeroth order amplitude) subtract from the transition rate. The Bremsstrahlung emission of real photons adds to the decay rate by introducing a new photon channel. However, the elastic subtraction wins out and determines the overall sign of the effect.

The rules of quantum electrodynamics for the diagrams in FIG.1 require the propagator (most often) written in the Feynman gauge. For the vacuum

$$D_{\mu\nu}^{vac}(\mathbf{k},\omega) = \left(\frac{4\pi\eta_{\mu\nu}}{|\mathbf{k}|^2 - (\omega/c)^2 + i0^+}\right). \quad (3)$$

In condensed matter, wherein resides the stopped muon, the properties of the medium will be described by a dielectric response function analytic in the upper-half complex frequency ζ plane; i.e.

$$\varepsilon(\zeta) = 1 + \frac{2}{\pi}\int_0^\infty \frac{\omega \Im m\ \varepsilon(\omega + i0^+)d\omega}{\omega^2 - \zeta^2}\ , \quad (4)$$

where $\Im m\ \zeta > 0$. In the condensed matter media, the Feynman gauge propagator $D_{\mu\nu}(\mathbf{k},\omega)$ is still diagonal in the indices $(\mu\nu)$. However, the diagonal elements are given by [10]

$$D_{00}(\mathbf{k},\omega) = -\left(\frac{(4\pi/\varepsilon(|\omega| + i0^+))}{|\mathbf{k}|^2 - \varepsilon(|\omega| + i0^+)(\omega/c)^2 + i0^+}\right) \quad (5)$$

and

$$D_{ij}(\mathbf{k},\omega) = \left(\frac{4\pi\delta_{ij}}{|\mathbf{k}|^2 - \varepsilon(|\omega| + i0^+)(\omega/c)^2 + i0^+}\right) \quad (6)$$

Eqs.(5) and (6) are required for the photon propagator insertions in the Feynman diagrams of FIG.1. For the Bremsstrahlung emission diagrams, the out going photon wave function must be renormalized in the following manner: (i) Define the condensed matter polarization part $\Pi^{\lambda\sigma}(\mathbf{k},\omega)$ via

$$D_{\mu\nu} = D_{\mu\nu}^{vac} + D_{\mu\lambda}^{vac}\Pi^{\lambda\sigma}D_{\sigma\nu}^{vac}. \quad (7)$$

(ii) If $a_\mu(\mathbf{k},\omega)$ denotes the outgoing photon wave function in the vacuum, and if $A_\mu(\mathbf{k},\omega)$ denotes the outgoing photon wave in the condensed media, then

$$A_\mu = a_\mu + D_{\mu\sigma}^{vac}\Pi^{\sigma\nu}a_\nu. \quad (8)$$

(iii) Finally, in the high frequency limit

$$\varepsilon(\zeta) \to 1 - \left(\frac{\omega_p^2}{\zeta^2}\right) \quad \text{as} \quad |\zeta| \to \infty \quad (9)$$

where the "plasma frequency" is determined by the number of electrons per unit volume n_e via

$$\omega_p^2 = \left(\frac{4\pi n_e e^2}{m_e}\right). \quad (10)$$

Equivalently we have the sum rule

$$\frac{2}{\pi}\int_0^\infty \omega \Im m\ \varepsilon(\omega + i0^+)d\omega = \omega_p^2, \quad (11)$$

so that $\hbar\omega_p$ sets a photon energy scale beyond which the condensed matter radiation impedance is the same as the vacuum radiation impedance. In practice, condensed matter effects are important only for photons of energy less than (say) 5 KeV.

For a stopped muon decay, the radiation given off by the product electron is substantial. To zeroth oder, the distribution of the product electron energy E is given by

$$d\Gamma_F(E) = \Gamma_F dP(E), \quad (0 < E < W), \quad (12)$$

where

$$\frac{dP(E)}{dE} = 6(\frac{E^2}{W^3}) - 4(\frac{E^3}{W^4}), \quad (13)$$

and the electron energy cut-off is $W = (M_\mu c^2/2)$. If $\mathbf{v}$ denotes the electron velocity,

$$E = \frac{m_e c^2}{\sqrt{1 - (|\mathbf{v}|/c)^2}}, \quad (14)$$

and $dN(\omega, E)$ denotes the distribution of Bremsstrahlung photons in a bandwidth $d\omega$, then for the vacuum

$$dN^{vac}(\omega, E) = \beta^{vac}(E)\left(\frac{d\omega}{\omega}\right) \quad (15)$$

where

$$\beta^{vac}(E) = \Big(\frac{\alpha}{\pi}\Big)\Big\{\Big(\frac{c}{|\mathbf{v}|}\Big)\ln\Big(\frac{c + |\mathbf{v}|}{c - |\mathbf{v}|}\Big) - 2\Big\}. \quad (16)$$

The corresponding distribution of radiated photons in the condensed matter environment

$$dN(\omega, E) = \beta(\omega, E)\left(\frac{d\omega}{\omega}\right) \quad (17)$$

has been discussed elsewhere [11]. The final result is given in terms of

$$z(\omega) = \Big(\frac{c}{|\mathbf{v}|\sqrt{\varepsilon(\omega + i0^+)}}\Big) \quad (18)$$

as

$$\beta(\omega, E) = \left(\frac{\alpha|\mathbf{v}|}{c\pi}\right)\left|\frac{(\Re e\ z)(\Im m\ \mathcal{G}(z))}{(\Im m\ z)}\right|, \quad (19)$$

where

$$\mathcal{G}(z) = \Big(\frac{z^2 - 1}{2}\Big)\ln\Big(\frac{z + 1}{z - 1}\Big) - z. \quad (20)$$

Using Eqs.(2), (5), (6), (13), (18), (19) and (20), and employing the condition that $\hbar\omega << W$ in the integral regime in which the material and vacuum values β differ appreciably, one finds

$$\eta = \left(\frac{2\pi}{\alpha}\right)\left(\frac{dP(E)}{dE}\right)_{E=W}\Delta E_{rad} \quad (21)$$

where

$$\Delta E_{rad} = \hbar \int_0^\infty \left(\beta(\omega, E = W) - \beta^{vac}(E = W)\right) d\omega. \quad (22)$$

Our central results follow from Eqs.(2), (13), (22) and (23). The total difference in the mean radiated energy $\Delta E_{rad} = E_{rad} - E_{rad}^{vac}$ between the material and the vacuum, at the maximum electron energy $W \approx (M_\mu c^2/2)$ determines the material radiation renormalization parameter

$$\eta = \left(\frac{8\pi}{\alpha}\right)\left(\frac{(E_{rad} - E_{rad}^{vac})}{M_\mu c^2}\right) \approx \left(\frac{\Delta E_{rad}}{30.78\ KeV}\right). \quad (23)$$

At this point it is important to distinguish three regimes for the motion of the charge. (i) When the muon is stopped, the velocity of the charge is zero. (ii) When the muon decays, there is a rapid acceleration of the charge to a final electron velocity $\mathbf{v}$. The acceleration produces a pulse of radiation with a mean photon distribution $\omega dN(\omega, E)/d\omega = \beta(\omega, E)$. These two regimes are present in the vacuum. (iii) The third (and final) regime is when the electron slowly decelerates with an energy loss as it leaves a track through the condensed matter medium. This last process is usually described by a retardation force [12]

$$F = -\left(\frac{dE}{dx}\right) \quad (24)$$

which has no counterpart in the vacuum.

For example, consider a regime in which the material is almost transparent; i.e.

$$\sqrt{\varepsilon(\omega + i0^+)} = n(\omega)e^{i\phi(\omega)}, \qquad |\phi(\omega)| << 1. \quad (25)$$

where n is the index of refraction and $\tan(2\phi)$ is the loss tangent of the dielectric response $\varepsilon(\zeta)$. In such a regime, and for

$$|\mathbf{v}| > \left(\frac{c}{n}\right) \qquad \text{(Cerenkov)}, \quad (26)$$

the Cerenkov radiation retardation force has the well known frequency distribution [13]

$$dF_C = \left(\frac{e^2}{c^2}\right)\left(1 - \left(\frac{c}{|\mathbf{v}|n}\right)^2\right)\omega d\omega. \quad (27)$$

This corresponds to the number of Cerenkov photons emitted in a bandwidth $d\omega$ and in a path length dx given by

$$\left(\frac{d^2 N_C}{d\omega dx}\right) = \left(\frac{\alpha}{c}\right)\left(1 - \left(\frac{c}{|\mathbf{v}|n}\right)^2\right). \quad (28)$$

Eq.(28) describes the emission of Cerenkov radiation during the third regime in which the electron leaves a track. The emission of Cerenkov radiation in the second regime (in which the charge rapidly accelerates from zero velocity to $\mathbf{v}$) is governed by Eqs.(18)-(20) and (25),

$$\beta_C = \left(\frac{\alpha|\mathbf{v}|}{2c\tan\phi}\right)\left(1 - \left(\frac{c}{|\mathbf{v}|n}\right)^2\right), \quad |\phi| << 1. \quad (29)$$

Note the loss angle term $tan\phi$ is in the denominator of Eq.(29). If the material in a frequency range of interest were really transparent, then $\phi \to 0$ would imply the divergence $\beta_C = \omega(dN_C/d\omega) \to \infty$. As previously discussed [11], the Cerenkov contribution to β thereby depends sensitively on the attenuation of the radiated waves. In some continuous media detector systems, e.g. optical fibres with a very small electromagnetic attenuation, there should be a very large flash of Cerenkov radiation.

The physical picture may be described as follows: Consider a modern jet plane leaving an airport. The plane starts out at rest, rapidly accelerates upon "take-off" and continues to accelerate until sound speed is approached. When the plane then accelerates right through the sound speed barrier, there follows a loud "thunder clap" or sound wave "explosion". (A considerate airline will have the pilot avoid breaking the sound speed barrier while flying over a city since the resulting *sound wave explosion* would be quite unpleasant for the the city inhabitants to hear). After the sound speed barrier is broken, the plane still sends out some further, but much more mild, sound waves along the "Mach-Cerenkov cone". The sounds are much more mild when flying at a uniform velocity than while accelerating through the sound speed barrier. With the sound wave analogy kept firmly in mind, let us return to the case of muon decay.

The muon starts off at rest and decays into an electron (plus, of course, uncharged objects). The electron quickly accelerates (so to speak) to a final velocity $\mathbf{v}$ which may break through the light speed barrier; i.e. $|\mathbf{v}| > (c/n)$ for some material bandwidths. When the light speed barrier is broken, there is an large flash of Cerenkov electromagnetic radiation with a photon distribution $(dN_C/N_C) = \beta_C(d\omega/\omega)$ according to Eq.(29). After the light speed barrier is broken, the electron (moving at a roughly uniform velocity) still sends out a further (but much more mild) Cerenkov electromagnetic signal in accordance with Eqs.(27) and (28).

The crucial point is the following: If the Cerenkov flash of radiation in the material is sufficiently large, then the electromagnetic renormalization of the muon decay rate *due to the material* will also be quite large. The exact numbers depend on the material properties $\varepsilon(\omega + i0^+)$.

For typical plastic coated glass optical fibres, we expect, in a frequency bandwidth $\hbar\Delta\omega \sim 0.2\ eV$, an index of refraction $n \sim 1.3$. Most importantly, there will be a small light wave attenuation (partly from Rayleigh scattering and partly from infrared electronic transitions) with a loss angle $\phi \sim 10^{-8}$ [14,15]. Using these estimates, we find for the Cerenkov contribution to Eq.(2) $\eta_C \sim 0.9$ which is substantial. For muon decay in metals, the effect is much reduced in magnitude. We note (in passing) that metals suppress radiation which may change the sign of η to a negative value.

While we were led to the notion of a Cerenkov flash in optical fibres via the consideration of the Feynman diagrams in FIG.1, it would appear to us to be of scientific interest to measure such optical "bolts of lightning" in situ per se. By contrast, when a charged pion at rest decays in a transparent medium, the Cerenkov flash will be very small or virtually zero. In pion decay the muon emerges at a velocity which can hardly beat that of light in the material. When the muon decays, the flash caused by the emerging electron (moving almost at vacuum light speed) should be substantial and measurable. These Cerenkov effects are within the technology of ongoing and planned precision muon decay experiments.

Finally, in a pioneering experiment [16], a dielectric suppression has been verified for Bremsstrahlung emission in scattering processes. This study gives us confidence in the reality of the material contributions to radiative corrections.

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