Muon Capture by Deuterons*

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Rates of muon capture from the separate μd hyperfine states are computed. The dependence of the measurement of effective coupling constants on the neutron energy is explicitly demonstrated. The calculation of the neutron energy spectra includes mainly the following refinements: (1) use of two-nucleon wave functions with hard core, (2) corrections for the target-proton momentum, and (3) inclusion of certain induced pseudoscalar terms. The capture rates obtained are 334 and 15 $\sec^{-1}$ for the μd doublet and quartet states, respectively.

I. INTRODUCTION

IN the previous paper[1] there is described an experimental attempt to study the capture of the muon by the deuteron. The detection efficiency for the neutrons emitted in the capture process had a lower threshold close to 2 MeV. It might be expected that with the advent of improved experimental techniques the μd capture process could be efficiently observed in the near future in a lower energy region (i.e., roughly from 1 to 3 MeV). The implication here, as will be clear, is the possibility to measure an almost pure Gamow-Teller coupling. With this hope, we have made some theoretical calculations on the μd capture process.

In the past there have been several papers written on this subject. To our knowledge, the article by Überall and Wolfenstein[2] is a relatively comprehensive one. Our attempt here has been motivated by both the analysis of experimental data and a renewed theoretical interest. The points we wish to emphasize are:

(1) Accurate neutron energy spectra for muon capture from the separate μd hyperfine states are necessary to make direct comparison with the data obtained from observations of muon capture in the $(p\mu d)^+$ molecules.[1]

(2) The recent observation of electrodisintegration of deuteron[3] provided some new information about the two-nucleon wave functions for both bound and unbound states. This is particularly relevant to processes with breakup of the two nucleons in the final state.

(3) Some refinements of the theory are made. They include mainly the relativistic target-proton velocity terms and the induced pseudoscalar terms in the hyperfine spectra.

(4) The effective coupling constant measured in a given energy region is dependent on certain "dynamical integrals." An explicit presentation of this relation is made.

II. MUON CAPTURE WITH NEUTRON EMISSION

Let us discuss very briefly the phenomenological muon-capture theory from which we will deduce a simple formula for muon capture with neutron emission. As is well known, the invariant Hamiltonian density is completely analogous to that in the Fermi theory of beta decay:

$$H=(G/\sqrt{2})[J_\lambda{}^N(J_\lambda{}^l)^\dagger+\text{H.c.}].$$

$J_\lambda{}^l=i\bar{\Psi}_\mu\gamma_\lambda(1+\gamma_5)\Psi_\nu$ is the leptonic current, and $J_\lambda{}^N=J_\lambda{}^V+J_\lambda{}^A$ denotes the weak nucleonic current. The general invariant form of the matrix element $\langle n|J_\lambda{}^N|p\rangle$ was first studied by Goldberger and Treiman[4] and by Weinberg.[5] If one makes the assumptions of conserved vector current and G-parity invariance, then the matrix elements of the vector and axial-vector currents can be written ($\hbar=c=1$)

$$\langle n|J_\lambda{}^V|p\rangle=i\bar{U}_n[F_V(q^2)\gamma_\lambda+F_M(q^2)i\sigma_{\lambda\rho}q_\rho]U_p,$$
$$\langle n|J_\lambda{}^A|p\rangle=i\bar{U}_n[F_A(q^2)\gamma_\lambda+F_P(q^2)iq_\lambda]\gamma_5U_p,$$
$$q_\lambda=p_\lambda-n_\lambda.$$

In Cabibbo's notation, these are related to the general Cabibbo currents[6,7] by the simple relation $f_{ABi}=d_{ABi}=1$. The coupling constant G is identical to that for muon decay. The form factors will then satisfy the following relations

$$F_V(0)=\cos\theta,\quad F_A(0)=-g_A/G\cong1.19\cos\theta,$$
$$F_M(q^2)=[(\mu_p-\mu_n)/2M]F_2(q^2)\cos\theta,$$
$$F_P(q^2)=[2M/(q^2+m_\pi{}^2)]F_A(q^2),\quad\text{with}\quad\cos\theta\cong0.975,$$

where g_A is the axial-vector coupling constant, F_2 is the nucleon magnetic-moment form factor. The q^2 dependence of the form factors is assumed[8] to be of the form $F(q^2)=F(0)(1-\frac{1}{6}\langle r^2\rangle q^2)$. Following Primakoff[9] the

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[1] I-T. Wang, E. W. Anderson, E. J. Bleser, L. M. Lederman, S. L. Meyer, J. L. Rosen, and J. E. Rothberg, this issue, Phys. Rev. **139**, B1528 (1965).

[2] H. Überall and L. Wolfenstein, Nuovo Cimento **10**, 136 (1958).

[3] H. W. Kendall, J. I. Friedman, E. F. Erickson, and P. A. M. Gram, Phys. Rev. **124**, 1596 (1961).

[4] M. L. Goldberger and S. B. Treiman, Phys. Rev. **111**, 355 (1958).

[5] S. Weinberg, Phys. Rev. **112**, 1375 (1958).

[6] N. Cabibbo, Phys. Rev. Letters **10**, 531 (1963).

[7] J. J. Sakurai, Phys. Rev. Letters **12**, 79 (1964).

[8] For a recent discussion of form factors and weak Cabibbo currents see N. Brene, B. Hellesen, and M. Roos, Phys. Letters **11**, 344 (1964). (Note, however, the difference in the definition of $\sigma_{\lambda\rho}$.)

[9] H. Primakoff, Rev. Mod. Phys. **31**, 802 (1959); A. Fujii and H. Primakoff, Nuovo Cimento **12**, 327 (1959).

above matrix elements and the leptonic current give an effective Hamiltonian of muon capture (for a system of A nucleons) in the two-component spinor representation:

$$H_{\text{eff}}=\sum_{i=1}^{A} H_i$$

$$=\tfrac{1}{2}\tau^{(+)}(1-\boldsymbol{\sigma}\cdot\hat{p})\sum_{i=1}^{A}\tau_i^{(-)}\delta(\mathbf{r}-\mathbf{r}_i)$$

$$\times[G_V{}^{\mu}+G_A{}^{\mu}(\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}_i)-G_P{}^{\mu}(\boldsymbol{\sigma}\cdot\hat{p})(\boldsymbol{\sigma}_i\cdot\hat{p})$$

$$-g_V{}^{\mu}(\boldsymbol{\sigma}\cdot\hat{p})(\boldsymbol{\sigma}\cdot\mathbf{p}_i)/M-g_A{}^{\mu}(\boldsymbol{\sigma}\cdot\hat{p})(\boldsymbol{\sigma}_i\cdot\mathbf{p}_i)/M], \quad (1)$$

where we have used the following definitions:

$$G_V{}^{\mu}\equiv G[F_V(1+\nu/2M)]\equiv g_V(1+\nu/2M),$$

$$G_A{}^{\mu}\equiv G[F_A-(F_V/2M+F_M)\nu]\equiv g_A{}^{\mu}-(g_V{}^{\mu}+g_M{}^{\mu})\nu/2M,$$

$$G_p{}^{\mu}\equiv G[(m_\mu F_P-F_A)\nu/2M-(F_V/2M+F_M)\nu]\equiv(g_P{}^{\mu}-g_A{}^{\mu}-g_V{}^{\mu}-g_M{}^{\mu})\nu/2M.$$

The last two terms in H_{eff} are the relativistic corrections linearly proportional to the initial target-proton momentum.

The physical processes we are concerned with belong to a rather general class of muon capture. They are characterized by the neutron emission in the final states. The transition matrix element for such reactions is related to the effective Hamiltonian H_{eff} by

$$M=\int F_b{}^*(\mathbf{r})\Psi_b{}^*\sum_{j=1}^{A}\exp(-i\boldsymbol{\nu}\cdot\mathbf{r}_j)\varphi_\mu(\mathbf{r}_j)H_j\Psi_a dV,$$

where dV is a $3(A-1)$-dimensional volume element. $F_b(\mathbf{r})$ describes the relative motion of the ejected neutron and the residual nucleus. φ_μ is the wave function of the muon around the initial nucleus. It is convenient to express the transition matrix element in the following form:

$$M=L_1N_1+\mathbf{L}_2\cdot\mathbf{N}_2+M_{\text{rel}},$$

where L stands for leptons and N for nucleons. Explicitly, they are

$$L_1=\tfrac{1}{2}(1-\boldsymbol{\sigma}\cdot\hat{p})G_V{}^{\mu},$$

$$\mathbf{L}_2=\tfrac{1}{2}(1-\boldsymbol{\sigma}\cdot\hat{p})[G_A{}^{\mu}\boldsymbol{\sigma}+G_P{}^{\mu}\hat{p}];$$

$$N_1=\int F_b{}^*\Psi_b{}^*\sum_j\tau_j{}^{(-)}\exp(-i\boldsymbol{\nu}\cdot\mathbf{r}_j)\varphi_\mu(\mathbf{r}_j)\Psi_a dV,$$

$$\mathbf{N}_2=\int F_b{}^*\Psi_b{}^*\sum_j\tau_j{}^{(-)}\exp(-i\boldsymbol{\nu}\cdot\mathbf{r}_j)\varphi_\mu(\mathbf{r}_j)\boldsymbol{\sigma}_j\Psi_a dV.$$

M_{rel} naturally contains all the relativistic effects of the target proton. If we further confine ourselves to processes with initially unpolarized muon and nucleus, then the probability of having a neutron emitted with its momentum in the range $\mathbf{p}_n$, and $\mathbf{p}_n+d\mathbf{p}_n$ is given by

$$d\omega=[2(2J_a+1)(2\pi)^5]^{-1}\times d\mathbf{p}_n\int d\boldsymbol{\nu}\sum_{\lambda,\lambda'}|M|^2\delta(E-\nu-E_n-E_b), \quad (2)$$

where

$$E=m_\mu+m_a-m_b-M_n,$$

$E_n=p_n{}^2/2M_n=$ kinetic energy of the emitted neutron,

$E_b=$ kinetic energy of the residual nucleus,

and λ stands for the totality of initial muon and nuclear-spin states, and λ' for that of the neutrino and final nuclear-spin states.

III. μd SPIN ORIENTATIONS AND CAPTURE RATES

Of the physical reasons for the important hyperfine effects discussed by Bernstein *et al.*,[10] the relative spin orientation of the proton and muon clearly plays the most vital role in the case of muon capture by deuteron. One expects a relatively much higher capture rate from the μd doublet state ($j=\frac{1}{2}$) than from the quartet state ($j=\frac{3}{2}$). It is well known that the high rate of muon capture by the $(p\mu p)^+$ molecular ion in pure hydrogen is due to the "ortho"-spin configuration. Still one would be very much interested to see how the presence of the neutron in the deuteron would affect the hyperfine capture rates. Probably the most remarkable feature of the μd capture is the Pauli exclusion principle effect.[9,11] As has been repeatedly pointed out, this effect is so large that the muon capture by the deuteron occurs almost exclusively in the Gamow-Teller mode; and we shall see later, this indeed provides an excellent chance to study the effective Gamow-Teller coupling.

For simplicity of notation, let us first confine ourselves to the main terms in the muon-capture matrix element. We will assume that the muon is initially unpolarized. The hyperfine states of the μd system are then completely specified by the following density matrices.[12]

$$\rho_{1/2}=\tfrac{1}{6}[1-\mathbf{s}\cdot\boldsymbol{\sigma}_\mu]P_d \quad \text{for} \quad j=\tfrac{1}{2},$$

$$\rho_{3/2}=\tfrac{1}{6}[1+\tfrac{1}{2}\mathbf{s}\cdot\boldsymbol{\sigma}_\mu]P_d \quad \text{for} \quad j=\tfrac{3}{2},$$

where $\mathbf{s}=\frac{1}{2}(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)$; $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$ are the Pauli spin matrices for the two nucleons. $P_d=\frac{1}{4}(3+\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)$ is the usual triplet spin projection operator for deuteron. With the help of these density matrices and the two-nucleon spin

[10] J. Bernstein, T. D. Lee, C. N. Yang, and H. Primakoff, Phys. Rev. **111**, 313 (1958). See also R. Winston, *ibid.* **129**, 2766 (1963).

[11] A. Rudik, Dokl. Akad. Nauk. SSSR **92**, 739 (1953); H. Primakoff, Rev. Mod. Phys. **31**, 802 (1959).

[12] I. M. Shmushkevich, Zh. Eksperim. i Teor. Fiz. **36**, 953 (1959) [English transl.: Soviet Phys.—JETP **9**, 673 (1959)]; A. P. Bukhvostov and I. M. Shmushkevich, Zh. Eksperim. i Teor. Fiz. **37**, 1042 (1960) [English transl.: Soviet Phys.—JETP **10**, 1471 (1960)].

projection operators, the transition matrix element $|M|^2$ can be averaged over all spin orientations of the μd hyperfine state and summed over all spin orientations of the neutrino and the final two neutrons. Explicit calculations of the traces are given in Appendix A. We will first state our results for the muon-capture rates (with one of the final neutrons in the energy range from E_n to E_n+dE_n) without relativistic corrections for the initial target-proton momentum.

(1) μd doublet case:

$$d\omega_{1/2}=[M_n{}^2\gamma_d/(2\pi)^3]\times\{[(G_V-2G_A)^2+\tfrac{2}{3}G_P(2G_V-4G_A+G_P)]I_t+\tfrac{1}{3}(3G_A-G_P)^2I_s\}dE_n. \quad (3)$$

(2) μd quartet case:

$$d\omega_{3/2}=[M_n{}^2\gamma_d/(2\pi)^3]\{[(G_V+G_A)^2-\tfrac{2}{3}G_P(G_V+G_A-G_P)]I_t+\tfrac{1}{3}G_P{}^2I_s\}dE_n. \quad (4)$$

(3) μd atom case:

$$d\omega=\tfrac{1}{3}d\omega_{1/2}+\tfrac{2}{3}d\omega_{3/2}=[M_n{}^2|\varphi_\mu(0)|^2/(2\pi)^3]\times\{G_V{}^2I_t+[G_A{}^2+\tfrac{1}{3}(G_P{}^2-2G_AG_P)](2I_t+I_s)\}dE_n. \quad (5)$$

In the above expressions, the coupling constants are understood to be the appropriate values averaged over the range of ν in I_i; M_n=neutron mass, γ_d=muon density at the position of the deuteron [in the $(p\mu d)^+$ molecular ion or in the μd atom]. We note that as the "induced coupling" effects are completely turned off (keeping the "renormalization" effect of g_A unchanged), $G_A\cong-1.19G_V$, $G_P\rightarrow 0$, and the rate of muon capture from the quartet state will become negligibly small, as is easily seen from Eq. (4). Although the hyperfine effects enter into the muon-capture rates as particular combinations of effective coupling constants, the final neutrino and two-neutron kinematics as well as the Pauli exclusion principle in the final state of interacting neutrons are involved in the integrals I_i (to be defined and studied in Sec. IV). In the following we will discuss these effects in connection with the two-nucleon wave functions.

IV. TWO-NUCLEON WAVE FUNCTIONS

The integral I_i that appears in Eqs. (3), (4), and (5) is defined as follows:

$$I_i=\int_{\nu_{\min}}^{\nu_{\max}}|J_i|^2\nu d\nu,$$

where

$$i=s(\text{singlet}),\quad t(\text{triplet}),$$

$$J_i=\int F_i{}^*(\mathbf{r},\mathbf{q})\exp(-i\boldsymbol{\nu}\cdot\mathbf{r}/2)\Psi_d(\mathbf{r})d\mathbf{r},\quad\text{with}\quad \mathbf{q}=\tfrac{1}{2}(\mathbf{p}_1-\mathbf{p}_2).$$

In Fig. 1 the I_s/I_t ratio is plotted versus the neutron energy. Comparing the figure with Eq. (3) one can see that the effective Gamow-Teller coupling is relatively enhanced in the 1- to 3-MeV region in a rather striking manner.

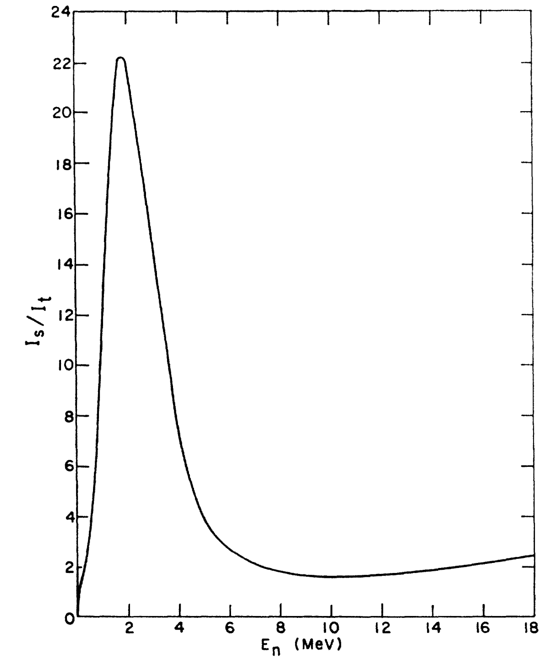

FIG. 1. I_s/I_t ratio versus neutron energy.

It is of interest to study the effects of the deuteron wave function Ψ_d and the unbound two-neutron wave function F_s on the muon-capture process. Recently, these wave functions were studied in relation to the electrodisintegration of deuteron.[3] The trick is to introduce a phenomenological two-nucleon interaction in the form of an Eckart potential,[13]

$$V(r)=-2\lambda^2/\cosh^2(\lambda r-\theta),$$

with a repulsive core of radius 0.42×10^{-13} cm. Both the ground-state solution Ψ_d and the S-state unbound solution $F(\mathbf{r},\mathbf{q})$ can be analytically obtained, and they give a very good fit of the data. The results indicate that the correct form of the two-nucleon wave function, in the conventional nonrelativistic approach, should contain a shift about the two-nucleon origin of the order of 0.42 F. A significant advantage of this approach is clearly the guaranteed orthogonality of the two-nucleon wave functions associated with different energy states and we believe that the present form of the deuteron wave function is capable of giving better accuracy for relatively small internucleonic distances which become important when large momentum transfer is involved in processes with breakup of the two nucleons in the final state.

In the actual computation we have adopted a three-term exponential type of wave function for the S-state part of the deuteron, and it is normalized to 0.97. [Be-

[13] C. Eckart, Phys. Rev. 34, 1303 (1930); V. Bargmann, Rev. Mod. Phys. 21, 488 (1949).

cause of the smallness of the $j_2(\frac{1}{2}\nu r)$ term in the neutrino plane wave, the deuteron D-state function itself has practically no effect on the capture rate.] It can be written as

$$\Psi_d=0, \qquad r\leq r_0$$
$$\Psi_d=Nr^{-1}\sum_{i=1}^{3}A_i\exp[-\alpha_i(r-r_0)], \quad r>r_0 \tag{6}$$

with

$$A_1=1, \quad A_2=-2(1-\gamma_t)^{-1}, \quad A_3=(1+\gamma_t)/(1-\gamma_t),$$
$$\alpha_1=\lambda_t\gamma_t, \quad \alpha_2=\lambda_t(2+\gamma_t), \quad \alpha_3=\lambda_t(3+\gamma_t),$$

and $\gamma_t=\tanh\theta_t$. $\boldsymbol{\lambda}_t$ and $\tanh\theta_t$ are parameters fitted to the empirical data of the triplet n-p scattering length and the effective range:

$$a_t=5.39\times10^{-13}\text{ cm}, \quad \rho_t=1.70\times10^{-13}\text{ cm}.$$

The above form of deuteron wave function turns out to be a very good approximation to the exact solution with Eckart potential. In the case of $r_0=0.42$ F, the following comparison can be made. For $r>3$ F, both the present form of deuteron wave function and that of Jankus[14] are so close to the exact solution (well within a tenth of one percent) that the difference becomes totally negligible. From 0.5 to 1 F, however, both approximate forms of wave function deviate from the exact solution by 1 to 3%. The present function is somewhat more accurate than the Jankus type in the region of $r=1$ to 3 F. Therefore, the difference in their accuracies is rather insignificant. But the function in Eq. (6) is considerably more convenient in actual calculations. In the special case of $A_3=0$, it can easily be reduced to the well-known Hulthen wave function in a computer program should such a comparison be desirable.

The final-state interaction of the two neutrons is included only in the singlet S state. The higher angular-momentum states have negligible final-state interactions because of the angular-momentum barriers; the contributions from these states are generally much smaller than from the S state. The Pauli principle enters in the space part of the two-neutron wave function, $F_s(\mathbf{r},\mathbf{q})$, in the usual manner

$$F_s(\mathbf{r},\mathbf{q})=(1/\sqrt{2})\{\exp(i\mathbf{q}\cdot\mathbf{r})+\exp(-i\mathbf{q}\cdot\mathbf{r})+2[Q_s(\mathbf{r},\mathbf{q})-j_0(qr)]\}.$$

In the above expression, $Q_s(\mathbf{r},\mathbf{q})=0$ for $r\leq r_0$; and,

$$Q_s(\mathbf{r},\mathbf{q})=(Ne^{-i\delta}/qr)\{C_1\sin(qr')+C_2\cos(qr')-\sum_{i=1}^{2}B_i\exp(-\beta_i r')[C_3\sin(qr')+q\cos(qr')]\},$$

for $r>r_0$ where $r'=r-r_0$, with

$$C_1=q^2-(\lambda_s^2\gamma_s), \quad C_2=(1+\gamma_s)\lambda_s q, \quad C_3=-\lambda_s\gamma_s;$$
$$B_1=2\lambda_s(1+\gamma_s)/(1-\gamma_s), \quad B_2=-\lambda_s(1+\gamma_s)^2/(1-\gamma_s);$$
$$\beta_1=2\lambda_s, \quad \beta_2=3\lambda_s.$$

The parameters λ_s and γ_s are again fitted to the experimental data[15] for two-neutron scattering length a_s and the effective range ρ_s:

$$a_s=-16.4\times10^{-13}\text{ cm},$$
$$\rho_s=2.65\times10^{-13}\text{ cm}.$$

With the above wave functions, the integrals J_s and J_t can be calculated analytically. The evaluation of I_s and I_t was then carried out by numerical integration with an IBM-7094 computer. As a natural consequence of the exclusion principle, I_t is generally much smaller than I_s.

V. CORRECTIONS FOR TARGET-PROTON MOMENTUM

The main contribution to the relativistic term M_{rel} in the transition matrix element comes from the products of $g_V{}^\mu$ and $g_A{}^\mu$ terms [Eq. (1)] with the main terms. Normally it amounts to only a few percent. However, its importance is relatively enhanced when the main terms are depressed due to Pauli exclusion principle effect. It is therefore interesting to see how its contribution would affect the μd hyperfine capture rates. The process of summation over the final-state and averaging over the initial-state spin orientations presents no new problem (Appendix B). Therefore we only need to consider here the factor $\boldsymbol{\sigma}_i\cdot\mathbf{p}_i$ in M_{rel}. Because of the complete antisymmetry of the two-nucleon wave functions in both the initial and the final states, it is sufficient to calculate a matrix element of the form $\langle b|\exp(i\boldsymbol{\nu}\cdot\mathbf{r}_1)\times(\boldsymbol{\sigma}_1\cdot\mathbf{p}_1)|a\rangle$. For simplicity we will write the complex conjugate of the space integral in two parts:

$$\left(\int d\mathbf{r}\right)^*=J_1+J_2,$$
$$J_1=\int\Psi_d{}^*[\hat{\nu}\cdot\mathbf{p}_1\exp(i\boldsymbol{\nu}\cdot\mathbf{r}_1)]F(\mathbf{r})d\mathbf{r}$$
$$=\nu\int\Psi_d{}^*\exp(i\boldsymbol{\nu}\cdot\mathbf{r}_1)F(\mathbf{r})d\mathbf{r}$$
$$J_2=\int\Psi_d{}^*\exp(i\boldsymbol{\nu}\cdot\mathbf{r}_1)(\hat{\nu}\cdot\mathbf{p}_1)F(\mathbf{r})d\mathbf{r}$$
$$\approx(\hat{\nu}\cdot\mathbf{p})\int\Psi_d{}^*\exp(i\boldsymbol{\nu}\cdot\mathbf{r}_1)F(\mathbf{r})d\mathbf{r},$$

[14] V. Z. Jankus, Phys. Rev. **102**, 1586 (1956).

[15] R. P. Haddock, R. M. Salter, Jr., M. Zeller, J. B. Czirr, and D. R. Nygren, Phys. Rev. Letters **14**, 318 (1965). An earlier calculation based on the value $a_s=-22\times10^{-13}$ cm was reported in: I-T. Wang, Columbia University Nevis Report No. 128, 1965 (unpublished).

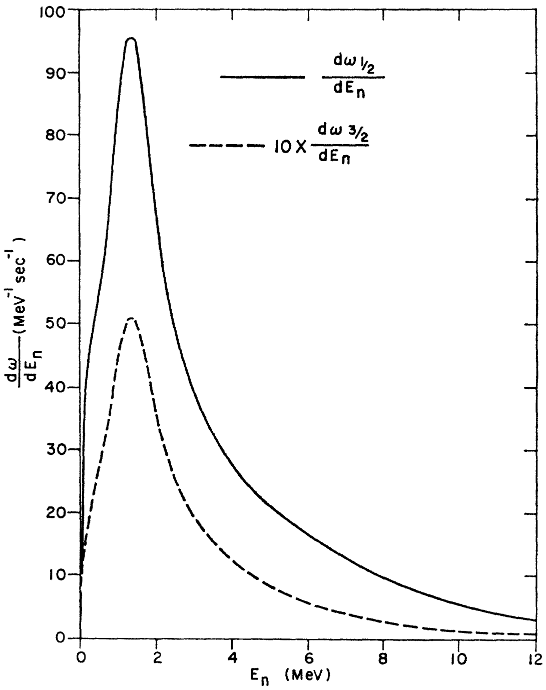

FIG. 2. Energy spectra of neutrons emitted from the muon-capture reactions initiated in the two separate μd hyperfine states.

where $\mathbf{p}=\mathbf{q}-\frac{1}{2}\boldsymbol{\nu}$. Therefore,

$$\int d\mathbf{r}=[\nu+(\hat{p}\cdot\mathbf{p})]\int F^*(\mathbf{r})\exp(-i\boldsymbol{\nu}\cdot\mathbf{r}_1)\Psi_d d\mathbf{r}.$$

In evaluating J_2 we have made the approximation that the final-state interaction is neglected. Actual calculations indicate that J_2 is significantly smaller than J_1, and therefore the over-all accuracy is essentially unaffected. Instead of the I_i integrals, the terms linearly proportional to the initial target-proton momentum will now contain the following integrals:

$$I_i'=\int_{\nu_{\min}}^{\nu_{\max}}|J_i|^2\nu^2 d\nu,$$

$$I_i''=\int_{\nu_{\min}}^{\nu_{\max}}|J_i|^2(\hat{p}\cdot\mathbf{p})\nu d\nu.$$

Combining the above results with the spin calculations in Appendix B, we can write the relativistic correction terms as

$$d\omega_{1/2}^{\mathrm{rel}}=[2M_n^2\gamma_d/3(2\pi)^3M_p]\{2[g_A(G_V+G_P-2G_A)+g_V(3G_A-G_P-\tfrac{3}{2}G_V)](I_t'+I_t'')+g_A(G_P-3G_A)(I_s'+I_s'')\}dE_n, \quad (7)$$

$$d\omega_{3/2}^{\mathrm{rel}}=[2M_n^2\gamma_d/3(2\pi)^3M_p]\{[g_A(2G_P-G_A-G_V)+g_V(G_P-3G_V-3G_A)](I_t'+I_t'')+g_AG_P(I_s'+I_s'')\}dE_n. \quad (8)$$

Again in the actual numerical computation the coupling constants are treated as functions of ν and integrated within I_i' and I_i''. The neutron energy spectra including the relativistic corrections are plotted in Fig. 2. These are for the μd atom case. The corresponding hyperfine capture rates obtained are

$$\omega_{1/2}=334\ \mathrm{sec}^{-1},\quad \omega_{3/2}=15\ \mathrm{sec}^{-1},$$

using $g_P{}^\mu\cong 7.2\ g_A{}^\mu$, and $g_A{}^\mu\cong -1.19\ (\cos\theta)G$. While the target-proton momentum amounts only to a 6% effect in the doublet capture rate, it is as high as 23% of the corrected rate in the case of capture from the quartet state.

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APPENDIX A: SPIN TRACE FOR THE μd CAPTURE—MAIN TERM

It is only necessary to present the spin-trace calculations for one of the μd hyperfine states. Let us consider, for example, the $j=\frac{1}{2}$ state. The pure Fermi term involved is extremely simple. It takes the form

$$G_V{}^2\,\mathrm{Tr}(\rho_{1/2}P_t)=\tfrac{1}{2}G_V{}^2.$$

The pure Gamow-Teller term is divisible into two parts. The factor associated with I_t is given by

$$\begin{aligned}\mathrm{Tr}[A_i^\dagger A_jP_t\sigma_{1j}\rho_{1/2}\sigma_{1i}]&=\tfrac{1}{3}[2G_A{}^2+(G_A-G_P)^2]-\tfrac{1}{3}[2G_AG_P-3G_A{}^2]\\&=\tfrac{1}{3}[6G_A{}^2-4G_AG_P+G_P{}^2],\end{aligned}$$

where

$$A_i=\tfrac{1}{2}(1-\boldsymbol{\sigma}\cdot\hat{\nu})[G_A\sigma_i-G_P(\boldsymbol{\sigma}\cdot\hat{\nu})\nu_i].$$

Similarly, the factor associated with I_s is

$$\begin{aligned}\mathrm{Tr}[A_i^\dagger A_jP_s\sigma_{1j}\rho_{1/2}\sigma_{1i}]&=\tfrac{1}{6}[2G_A{}^2+(G_A-G_P)^2]-\tfrac{1}{3}[2G_AG_P-3G_A{}^2]\\&=\tfrac{1}{6}[9G_A{}^2-6G_AG_P+G_P{}^2].\end{aligned}$$

As a check we will note that the above trace calculations involve the following nucleon parts:

$$\mathrm{Tr}n[P_t\sigma_{1i}\rho_{1/2}\sigma_{1j}]=\tfrac{1}{6}(2\delta_{ij}+i\epsilon_{ijk}\sigma_k);$$

$$\mathrm{Tr}n[P_s\sigma_{1i}\rho_{1/2}\sigma_{1j}]=\tfrac{1}{6}(\delta_{ij}+i\epsilon_{ijk}\sigma_k).$$

Finally, the cross term appears only with I_t and is given by

$$2G_V\,\mathrm{Tr}[A_i^\dagger P_t\rho_{1/2}\sigma_{1i}]=-\tfrac{2}{3}G_V(3G_A-G_P).$$

APPENDIX B: SPIN TRACE FOR THE TARGET-PROTON MOMENTUM TERM

The calculations involved are completely similar to those in Appendix A. For convenience we will consider separately the contributions for the g_A and g_V terms.

(1) g_A term: The contribution to the triplet state of two neutrons can be written as

$$(2g_A/M)\,\mathrm{Tr}[\mathfrak{M}\rho_{1/2}(\boldsymbol{\sigma}_1\cdot\mathbf{p}_1)P_t] = (2g_A/3M)(G_P-2G_A+G_V)(\hat{\nu}\cdot\mathbf{p}_1);$$

likewise, the corresponding term for the two-neutron singlet state is

$$(2g_A/M)\,\mathrm{Tr}[\mathfrak{M}\rho_{1/2}(\boldsymbol{\sigma}_1\cdot\mathbf{p}_1)P_s] = (g_A/M)(\tfrac{1}{3}G_P-G_A)(\hat{\nu}\cdot\mathbf{p}_1).$$

(2) g_V term: Only the two-neutron triplet state contributes:

$$(2g_V/M)\,\mathrm{Tr}[\mathfrak{M}\rho_{1/2}L(\boldsymbol{\sigma}\cdot\mathbf{P}_1)P_t] = (g_V/M)(\tfrac{1}{3}G_P-G_A-G_V)(\hat{\nu}\cdot\mathbf{p}_1).$$

In the above, M = proton mass, $L=\frac{1}{2}(1-\boldsymbol{\sigma}\cdot\hat{\nu})$, and

$$\mathfrak{M}=\tfrac{1}{2}(1-\boldsymbol{\sigma}\cdot\hat{\nu})[G_V+G_A(\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}_1)+G_P(\hat{\nu}\cdot\boldsymbol{\sigma}_1)].$$

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Study of the "Breakup" Channels of Muon Capture by He^3†

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Neutron energy spectra are obtained for the processes of muon capture by He^3 into the "breakup" channels: $\mu+He^3\to\nu+n+d$ and $\mu+He^3\to\nu+2n+p$. The chief approximations are the use of plane waves for the relative motions between the final nuclear particles and a Gaussian-type wave function for the He^3 nucleus. The rates obtained for the two "breakup" channels are, respectively, 988 and 272 sec^{-1}.

I. INTRODUCTION

The process of muon capture by He^3 into H^3 and neutrino has been the object of a great deal of experimental and theoretical interest. This is due, in part, to the relatively simple and fundamental nature of the He^3 nucleus and in part to the unique feature of the recoil triton that lends itself to refined experimental observations. Much less, however, has been done about the competing channels of muon capture by He^3. In this paper we wish to describe a brief effort to study these "breakup" channels:

$$\mu+He^3\to\nu+n+d,$$
$$\to\nu+2n+p.$$

It is not difficult to recognize the complexity of the nuclear physics involved in these reactions. Some of the problems are still far from being thoroughly understood. Consequently, the calculations we have made can only be said to be preliminary in nature, but we believe they help to reveal a few interesting features of the problem. In the meantime it is possible, with the help of the calculated neutron energy spectra, to make a comparison between the theory and the observed neutron rate. Such a comparison is reported in a separate paper.[1]

Aside from the practical aspects of the problem, there is an intrinsic theoretical interest in such calculations. Since the nuclear system involved is a composite of only three nucleons, it is still possible to compute the partial capture rates to all the final nuclear states without using the closure approximation. It would be interesting to see how the total capture rate so obtained compares with the closure approximation results, especially in view of the difficulty to provide a direct theoretical justification for the closure approximation.[2]

II. A COMPARISON BETWEEN THE "BOUND" AND "UNBOUND" CAPTURES

To understand the process of muon capture by the He^3 nucleus into a continuum state of neutron and deuteron, we find it helpful to illustrate it against the background of well-known theories of muon capture with bound final nuclear states.[3] In these latter cases, particularly muon captures by light nuclei, there can be stated some general physical principles. Among them, the following are of special interest to us. There can be no mixing between different orbital-angular-momentum states of the emitted neutrino waves (assuming no polarization in the initial muonic atom), a principle that follows from the absence of a physical mechanism

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